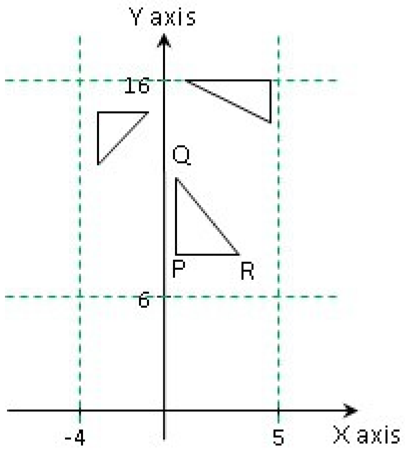
The key to doing well on GMAT Quant is to pick your standard questions, understand them really well and then try out variations of these questions. A small change in the question might force you to rethink your entire approach. The more you experiment, the more interesting your GMAT preparation will get and not to forget, the stronger your Quant will be. Let me show you what I mean with the help of an example. Since we have been doing co-ordinate geometry for the past few weeks, let's look at an interesting Official Guide question on coordinate geometry. Thereafter, we will try and figure out some variations of the same. We will focus more on the variations and how to think dynamically to arrive at solutions in those cases.

Official Guide Question: Right triangle PQR is to be constructed in the xy-plane so that the right angle is at P and PR is parallel to the x-axis. The x and y coordinates of P, Q and R are to be integers that satisfy the inequalities $-4 \leq x \leq 5$ and $6 \leq y \leq 16$. How many different triangles with these properties can be constructed?
(A) 110
(B) 1100
(C) 9900
(D) 10000
(E) 12100
Solution:
The triangle should be right at P and PR should be parallel to x axis. Also, the vertices P, Q and R should lie within or on the boundary of the green dotted rectangle shown in the diagram.
The triangle can take various orientations as shown.



Consider the green dotted rectangle. The x coordinate varies from -4 to 5 and the y coordinate varies from 6 to 16.
PR has to be parallel to x axis. Since it is a right angled triangle, PQ will be parallel to y axis.
Now let's first fix the vertex P. In how many ways can you choose the coordinates of P? The x coordinate of P can vary from -4 to 5 i.e. it can take 10 values. The y coordinate of P can vary from 6 to 16 i.e. it can take 11 values. In all, the coordinates of P can take 10*11 = 110 values.
Once vertex P is fixed, the x coordinate of Q will be the same as the x coordinate of P (since PQ is parallel to y axis) and the y coordinate of R will be the same as the y coordinate of P (since PR is parallel to x axis).
The y coordinate of Q can be chosen in 10 ways. (Out of the 11 values of y coordinate, one is occupied by P so 10 are leftover.)
The x coordinate of R can be chosen in 9 ways. (Out of 10 values of x coordinate, one is occupied by P so 9 are leftover.)
Total number of ways of making the triangle = 110 * 10 * 9 = 9900
Note that we have counted all possible triangles of all orientations in this case.
Answer: C
Hope you understand the method discussed. Here we were focusing on right triangles only. That made our task a little easier. What if we remove this condition? What if we had to find the total number of triangles? Let's try a variation without the right triangle condition.

Variation 1: How many triangles with positive area can be drawn on the coordinate plane such that the vertices have integer coordinates (x,y) satisfying 2 ≤ x ≤ 4 and 5 ≤ y ≤ 7?

(A) 72

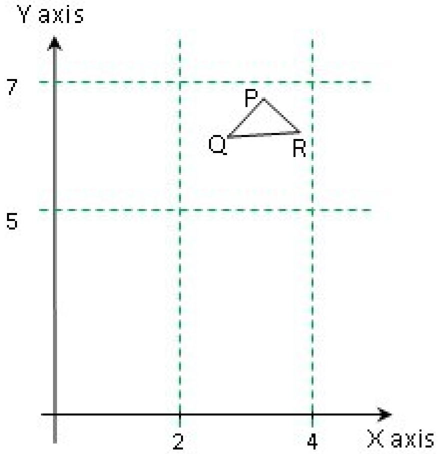
(B) 76

(C) 78

(D) 80

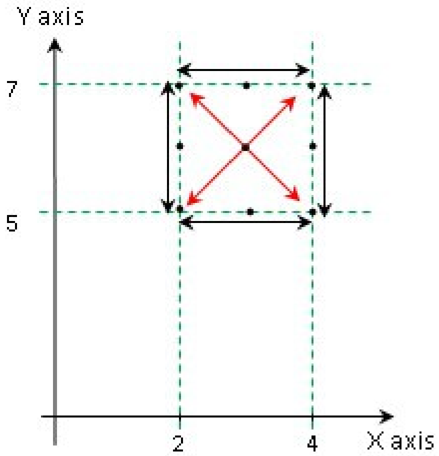
(E) 84

Let's draw the figure first.



With the given conditions, 2 ≤ x ≤ 4 and 5 ≤ y ≤ 7, the triangle should lie in the green dotted rectangle, as shown. For a vertex, the x coordinate can be chosen in 3 ways (2, 3 or 4) and the y coordinate can be chosen in 3 ways (5, 6 or 7). Hence we can choose a vertex in 3*3 = 9 ways (e.g. (2, 5), (2, 6), (2, 7), (3, 5) etc). We need 3 vertices to form a triangle. The first vertex can be chosen in 9 ways, the second vertex can be chosen in 8 ways and the third vertex can be chosen in 7 ways. Hence, there are a total of 9*8*7 ways of choosing the three vertices. But since we have ordered the vertices as first, second and third in this case, we need to divide this number by 3! (If this is unclear, don't worry. I will take up Permutations and Combinations next where we will discuss these concepts in detail.)

But these 9*8*7/3! cases also include those where all three vertices selected are collinear.



3 sets of horizontally collinear points: (shown by 2 horizontal black arrows)

(2, 5), (3, 5) and (4, 5)

(2, 6), (3, 6) and (4, 6)

(2, 7), (3, 7) and (4, 7)

3 sets of vertically collinear points: (shown by 2 vertical black arrows)

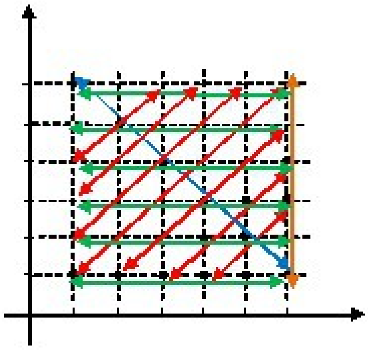
(2, 5), (2, 6), (2, 7)

(3, 5), (3, 6), (3, 7)

(4, 5), (4, 6), (4, 7)

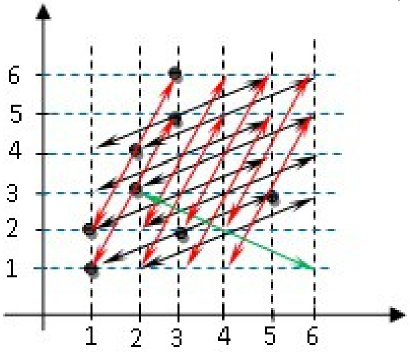
2 sets of diagonally collinear points: (shown by 2 red arrows)

(1, 1), (2, 2), (3, 3)
(1, 3), (2, 2), (3, 1)
We need to subtract these from 9*8*7/3! because collinear points do not make a triangle.
All other sets of 3 points will make a triangle.
Total number of triangles = 84 – 8 = 76
Answer (B)
We used brute force method in this question. We counted the number of collinear points and subtracted them from the total available number of coordinates. The problem of collinear points did not occur in the previous question because we were considering just right triangles there. What happens in case the acceptable region i.e. the dotted green rectangle is much larger? How will we calculate all the collinear points in that case? Let's see.
Variation 2: How many triangles with positive area can be drawn on the coordinate plane such that the vertices have integer coordinates (x,y) satisfying 1 ≤ x ≤ 6 and 1 ≤ y ≤ 6?
In this question, there are a total of 36 co-ordinates to choose from. We need to make triangles so we need to select a triplet of co-ordinates out of these 36 which can be done in 36*35*34/3! ways (same logic as above). Out of these, we need to get rid of those triplets where the points are collinear. How many such triplets are there? There are many sets which are horizontally, vertically and diagonally collinear. How do we count all these?
Look at the diagram given below:



There are six horizontally collinear points for each y co-ordinate shown by the green lines. From each of these 6 horizontal collinear points, you can select any 3 and they will not give you a triangle. You can select 3 points out of 6 in 6*5*4/3! ways (or 6C3). There are 6 such groups of 6 collinear points so we can select 3 collinear points (horizontally) in 6*6*5*4/3! ways = 120 ways.
Same is the case with vertically collinear points. There are six vertically collinear points for each x co-ordinate shown by the single orange line. From each of these 6 vertically collinear points, you can select any 3 and they will not give you a triangle. You can select 3 points out of 6 in 6*5*4/3! ways (or 6C3). There are 6 such groups of 6 collinear points so we can select 3 collinear points (vertically) in 6*6*5*4/3! ways = 120 ways.
Now let's look at the diagonally collinear points. The red arrows show the diagonally collinear points. You can select 3 points from the 3 collinear points in 1 way (shown by the smallest red arrow). You can select 3 points from a set of 4 collinear points in 4*3*2/3! = 4 ways. You can select 3 points from a set of 5 collinear points in 5*4*3/3! = 10 ways. You can select 3 points from a set of 6 collinear points in 6*5*4/3! = 20 ways. Now again you have 5 collinear points so you can select 3 out of them in 10 ways. Now you have 4 collinear points so you can select 3 out of them in 4 ways and from the remaining 3 collinear points, you can select 3 in 1 way.
In all, you can select 3 diagonally collinear points in 1+4+10+20+10+4+1 = 50 ways in this direction.
Now consider the opposite alignment shown by the blue arrow. Here again, you can select 3 collinear points in 50 ways (just like above)
Now consider collinear points lying on lines which have a slope of 2 or 1/2 i.e. for a change of 1 unit of one co-ordinate, the other co-ordinate changes by 2.

Look at the red arrows below. When x co-ordinate changes by 1, the y co-ordinate changes by 2. The points (1, 1), (2, 3) and (3, 5) lie on same line i.e. they are collinear. You have 8 red arrows and 8 black arrows. Similarly, you will have another set of 16 from the other end (shown by the green arrow)



Total sets of 3 collinear points = 120 + 120 + 50 + 50 + 32 = 372

Therefore, total number of triangles we can make in this case = 36*35*34/3! – 372

Each of the two variations required a twist to your strategy. The first variation was simpler since you just had to manually calculate the number of collinear points. The second one needed a little more thinking. I hope this example will encourage to try out some variations of the next interesting Quant question you come across.